Week 4 (disc 104)

Wednesday, February 10, 2021 10:49 AM

Math 128a – Week 4 Worksheet
GSI: Izak, (2/10/21)

2.3 Problems

Problem 1. *Come up with a function $f \in C^2[a,b]$ with $f(p) = 0$ for some $p \in [a,b]$ such that Newtons method fails to converge for any initial guess not equal to p.*

Problem 2. *Derive the error formula for Newton's method:*

$$|p - p_{n+1}| \leq \frac{M}{2|f'(p_n)|}|p - p_n|^2$$

2.4 Problems

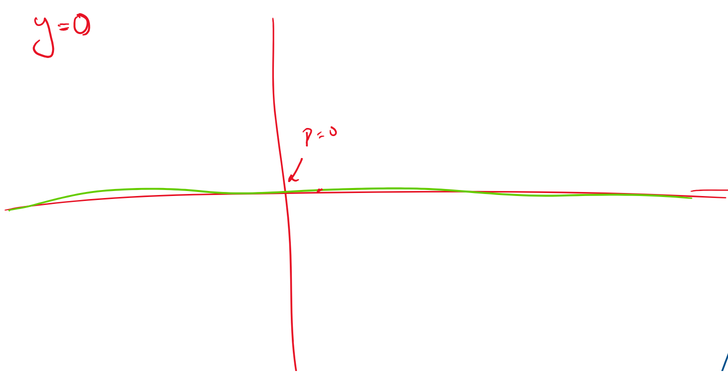
Problem 3. *Generalize one of your homework problems. Construct a sequence p_n converging to p at order α with asymptotic error constant λ.*

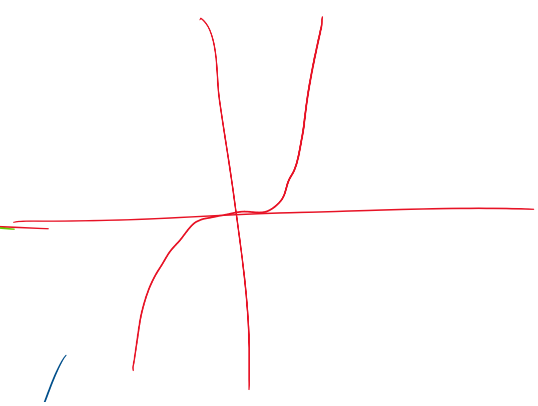
2.5 Problems

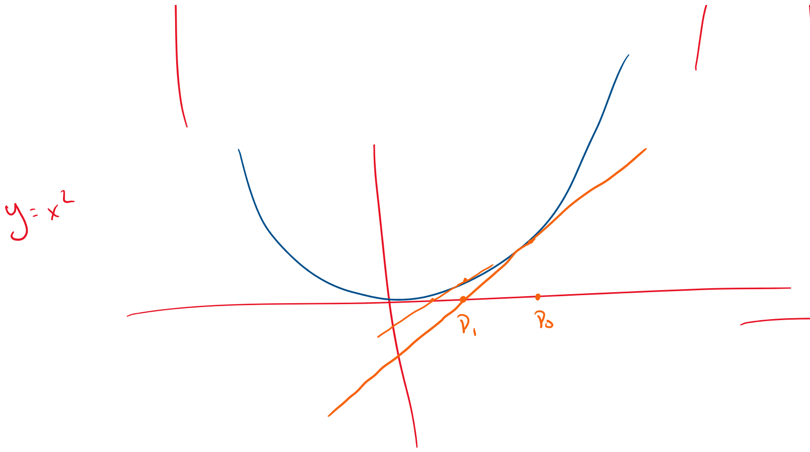
Problem 4. *Steffensen's method is applied to a function $g(x)$ using $p_0^{(0)} = 1, p_2^{(0)} = 3$ to obtain $p_0^{(1)} = .75$. What is $p_1^{(0)}$?*

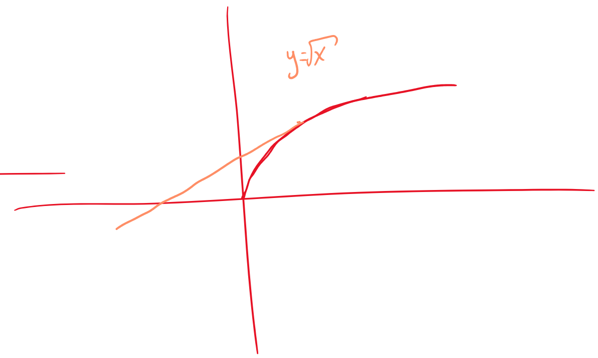
Problem 5. *Prove that if p_n converges linearly to p and $\lim_{n\to+\infty} \frac{p_{n+1}-p}{p_n-p} < 1$, then $\lim_{n\to+\infty} \frac{\hat{p}_n - p}{p_n - p} = 0$ where $\hat{p}_n$ comes from Aitken's Δ^2 method. (Hint: let $\delta_n = (p_{n+1} - p)/(p_n - p) - \lambda$ and show that $\lim_{n\to+\infty} \delta_n = 0$. Then express $(\hat{p}_{n+1} - p)/(p_n - p)$ in terms of δ_n, δ_{n+1} and λ).*

1) $y=0$









2) $|p - p_{n+1}|$

$$p_{n+1} = p_n - \frac{f(p_n)}{f'(p_n)} \qquad \text{Newton's method.}$$

$$|p_n - p_{n+1}| = \left| \frac{f(p_n)}{f'(p_n)} \right|$$

Assuming $f \in C^2$ $\quad f(p)=0 \quad f'(p) \neq 0$

Taylor about $x=p$

$$f(x) = \underset{0}{f(p)} + f'(p)(x-p) + \frac{f''(\xi)(x-p)^2}{2}$$

$$= f'(p)(x-p) + \frac{f''(\xi)(x-p)^2}{2}$$

@ $x = p_n$

$$f(p_n) = f'(p)(p_n - p) + \frac{f''(\xi)(p_n-p)^2}{2}$$

$$\frac{f(p_n)}{f'(p_n)} = \frac{f'(p)(p_n-p)}{f'(p_n)} + \frac{f''(\xi)(p_n-p)^2}{2f'(p_n)}$$

$$\left|\frac{f(p_n)}{f'(p_n)}\right| \leq \underbrace{\left|\frac{f'(p)(p_n-p)}{f'(p_n)}\right|}_{\text{need this zero.}} + M\frac{|p_n-p|^2}{2|f'(p_n)|}$$

if $f'(p)=0 \Rightarrow$ we get $\quad = 0$

we have our solution

Thm 2.9 in book.

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(3) $P_n = \lambda^n 10^{-\alpha^n}$ $\qquad P = 0.$

$$\frac{|P_{n+1} - P|}{|P_n - P|^\alpha} = \frac{10^{-\alpha^{n+1}}}{(10^{-\alpha^n})^\alpha} \to 1 \xrightarrow{?} \lambda$$

$$\frac{\lambda^{n+1} 10^{-\alpha^{n+1}}}{\lambda^{n\alpha} 10^{-\alpha^{n+1}}} \qquad \cancel{\lim_{n\to\infty} \frac{C_{n+1} 10^{-\alpha^{n+1}}}{C_n}}$$

$$\lim_{n\to\infty} \frac{C_{n+1}}{C_n^\alpha} = \lambda$$

$P = 0$ $\qquad \dfrac{P_{n+1} - P}{(P_n - P)^\alpha} = \lambda$ $\qquad \boxed{\lambda < 1}$

$\to P_{n+1} - P = \lambda (P_n - P)^\alpha$

$P_1 = 1$

$P_2 = \lambda (P_1)^\alpha = \lambda$

$P_3 = \lambda (P_2)^\alpha = \lambda(\lambda)^\alpha = \lambda^{\alpha+1}$

$P_4 = \lambda (P_3)^\alpha = \lambda(\lambda^{\alpha+1})^\alpha = \lambda(\lambda^{\alpha^2+\alpha}) = \lambda^{\alpha^2+\alpha+1}$

$P_5 = \ldots = \lambda^{\alpha^3+\alpha^2+\alpha+1}$

$$P_n = \lambda^{\sum_{i=0}^{n-2} \alpha^i}$$